**Lesson 0**

<https://math.libretexts.org/Courses/Monroe_Community_College>

Set Theory

**Subset**

if and only if

Set A is a subset of set B, denoted by , if every element of A is also an element of B.

To prove

1. Let be an arbitrary element of set .

2. Show is an element of set .

**Equal Sets**

Transitivity of subsets: Let , , and be sets. If and , then .

To prove sets equal

1. Show that

2. Show that

**Proper Subset**

The set is a proper subset of , denoted , if is a subset of , and .

For any set , we have and . In particular, .

**Powerset**

The set of all subsets of is called the power set of , denoted

From MAT2612:

Example: is the poset a lattice?

Therefore, is a lattice

Remember that:  
 *number of elements in*

*number of elements in the empty set*

*number of elements in the powerset*

*number of elements in the empty set*

Also:

. *every set is a subset of itself*

**Lesson 0**

<https://math.libretexts.org/Courses/Monroe_Community_College/MATH_220_Discrete_Math/5%3A_Functions/5.2%3A_De%EF%AC%81nition_of_Functions>

Functions

**Function**

Let and be nonempty sets. A function from to is a rule that assigns to every element of a unique element in .

is the domain of the function

is the codomain of the function

If the function is called , we write . Given , its associated element in is called its image.

*for every element in the domain, there exists a unique image in the codomain*

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|  | 0 | 3 | 1 | 4 | 2 | 0 | 3 |

**A Function as a Set of Ordered Pairs**

A function can be written as a set of ordered pairs from such that .

**Identity Function**

The identity function on a nonempty set

Maps any element back to itself

**Inverse function**

Let and be nonempty sets. function is said to be invertible if it has an inverse function.

Notation: If is invertible, we denote the (unique) inverse function by

<https://sites.math.washington.edu/~arms/m300A17/InvFunc.pdf>

<https://sites.math.washington.edu/~arms/m300A17/InvFunc.pdf>

Proof:

Suppose is an invertible function.

Then= a for every ;

= b for every ;

**One-to-One (Injection)**

A function is said to be one-to-one if *an injection*

A function that is not one-to-one is referred to as many-to-one.

To prove a function is One-to-One

1. Assume

2. Show it must be true that

3. Conclude: we have shown if then , therefore is one-to-one, by definition of one-to-one.

To prove a function is NOT One-to-One

1. exhibit one case (a counterexample) where and

2. Conclude: we have shown there is a case where and , therefore is NOT one-to-one.

Example: Prove the function defined by is one-to-one.

Assume , which means .

Thus

so .

We have shown if then . Therefore is one-to-one, by definition of one-to-one

Example: Prove the function defined by is NOT one-to-one.

Consider and .

Clearly .

However, and so .

We have shown there is a case where and , therefore is NOT one-to-one.

**ONTO (Surjection)**

A function is onto if, for every element , there exists an element such that

An onto function is also called a surjection, and we say it is surjective.

Proof:

For

Let be any element in the codomain, .

Figure out an element in the domain that is a preimage of ; often this involves some "scratch work" on the side.

Choose the value you found.

Demonstrate is indeed an element of the domain, .

Show .

Conclude with: we have found a preimage in the domain for an arbitrary element of the codomain, so every element of the codomain has a preimage in the domain. Therefore is onto, by definition of onto.

**One-to-One correspondence**

A function is said to be one-to-one correspondence if and only if is both:

Injective (one-to-one): and,

Surjective (ONTO): for all there is some such that

**Example:** <https://math.libretexts.org/Courses/Monroe_Community_College/MATH_220_Discrete_Math/5%3A_Functions/5.3%3A_One-to-One_Functions>

**Injectivity:**

Take and assume that

Thus

And

So

We have shown if then . Therefore is one-to-one, by definition of one-to-one.

**Surjectivity:**

We need to find an that maps to .

Suppose ;

Now we solve for in terms of .

We find

Proof:

Let be any element of .

Thus, we have found an such that

**Lesson 0**

<https://math.libretexts.org/Courses/Monroe_Community_College/MATH_220_Discrete_Math/5%3A_Functions/5.4%3A_Onto_Functions_and_Images%2F%2FPreimages_of_Sets>

Onto Functions and Images/Preimages of Sets

**Image of a Set**

Given a function , and , the image of under is defined as

*is the set of all the images of the elements of*

**Preimage of a Set**

Given a function , and , the image of under is defined as

*is the set of all the images of the elements of*

**Lesson 0**

Matrices

Rank and nullity

Example: JUNE 2016 Q1 8

Graphical user interface, text

Description automatically generated with medium confidence

wolframalpha

{{1,1,-1},{0,1,-1},{1,0,0},{1,-2,2}}

*This is how you input a matrix in wolfram*

Example: JUNE 2016 Q3.1

Let

Determine the nullity of

[1] Find

Wolframalpha

rref {{1,1,0},{0,1,0},{1,1,0}}

There are 2 nonzero rows in the row echelon form of the matrix, so the rank is 2

[2] No of columns – rank A

There are 3 columns and the rank 2

Example: JUNE 2016 Q3.2

Determine the characteristic equation for the eigenvalues of

Wolframalpha

{{1,1,0},{0,1,0},{1,1,0}}

*this is the row with the most zeros. We use this for the determinant*

Now we use the term without the zeros

Example: JUNE 2016 Q3.3

Determine the bases for the eigenspaces of

Wolframalpha

{{1,1,0},{0,1,0},{1,1,0}}

Characteristic equation

Eigenvalues

or

Remember that

So

[1] Substitute 0 for

*Use your first eigenvalue here*

Write he system as a system of scalar equations (i.e. reduce the augmented matrix)

[2] Substitute 1 for

*Use your second eigenvalue here*

Write he system as a system of scalar equations (i.e. reduce the augmented matrix)

Example: JUNE 2016 Q3.4

Is A diagonalizable?

Characteristic equation

Eigenvalues

or

Geometric multiplicity of

Algebraic multiplicity of

Geometric multiplicity of

Algebraic multiplicity of

Example:

<https://yutsumura.com/how-to-find-a-basis-for-the-nullspace-row-space-and-range-of-a-matrix/>

Let

[1] Find the basis for the nullspace of

Wolframalpha

rref {{1,0,9,2}, {0,1,-3,1}, {0,0,0,0}}

*This is the general solution of*

Compute general solution

Vector form solution to

It follows that the nullspace of matrix is given by

[2] Find the basis for the row space of

*This is basically*

*Also, one of the subspaces of*

**Lesson 0**

Vector Spaces

**Vector Language: The axioms for a vector space**

**Addition**

|  |  |
| --- | --- |
| is in | **Closure under addition** |
| *Vector addition is commutative and associative* | **Commutative property** |
|  | **Associative Property** |
| has a zero vector such that for every in , | **Associative Identity** |
| For every in , there is a vector in denoted by such that  *denotes the additive inverse of ;*  *Adding v to (−1)v gives the zero vector.* | **Additive inverse** |

**Scalar Multiplication**

|  |  |
| --- | --- |
| is in | **Closure under scalar multiplication** |
|  | **Distributive property** |
| *scalar multiplication is distributive* | **Distributive Property** |
|  | **Associative Property** |
| *Multiplying a vector v by the scalar 1 doesn’t change v* | **Scalar identity** |

Example: NOV 2016 Q1.1

Consider the set

*“X is assigned the set ”*

Operations

and Form a vector space

What statements are true in ?

Inner product

Example: JUNE 2016 Q 2.1

Show that

is an inner product on

[1] For all

[2] For all and

[3] For all

[4] Let . Then

And if and only if (since )

**span**

Adding up scalar multiples of vectors in a list gives what is called a linear

combination of the list

A vector space is called finite-dimensional if some list of vectors in it

spans the space

**Text

Description automatically generated**

**Text, letter

Description automatically generated**

**Usual operations of vector spaces**

Which of the following are subspaces of with the usual operations ?

Which of the following are subspaces of with the usual operations ?

*Remember, is the set of 2x2 matrices. The actual numbers don’t matter*

Example

<https://math.emory.edu/~lchen41/teaching/2020_Fall/Slides_6-2-Handout.pdf>

**Let**

, , ,

Show that

[1] First, show that

Proof: **Let**

, , ,

Since

and

if follows that

Proof: **Show that , can be written as**

***a linear combination of***

*TODO*

[2] Then show that

**linear independence**

**Text

Description automatically generated**

If some vectors are removed from a linearly independent list, the remaining

list is also linearly independent

Evaluate vectors in :

- Orthogonalize a vector (put it into matrix form)

- Calculate the determinant of the vector

- The vector is linearly independent if is non-zero

Test

- Two vectors and are linearly independent if the numbers

are the only ones satisfying

and

then is equivalent to

Example:

Are the vectors

,

, and

linearly independent?

Wolframalpha

det {{1,0,1}, {0,1,0}, {1,1,-1}}

Therefore, the vectors are linearly independent

Evaluate vectors in :

*Usually for vectors in spaces or higher*

- Use gram-Schmidt process. Method for orthonormalizing a set of vectors

<https://www.emathhelp.net/calculators/linear-algebra/gram-schmidt-calculator/>

Example: JUNE 2016 Q2.3

[1] Find orthogonal basis with respect to inner product

**bases**

**dimension**