**Lesson 0**

<https://math.libretexts.org/Courses/Monroe_Community_College>

Set Theory

**Subset**

if and only if

Set A is a subset of set B, denoted by , if every element of A is also an element of B.

To prove

1. Let be an arbitrary element of set .

2. Show is an element of set .

**Equal Sets**

Transitivity of subsets: Let , , and be sets. If and , then .

To prove sets equal

1. Show that

2. Show that

**Proper Subset**

The set is a proper subset of , denoted , if is a subset of , and .

For any set , we have and . In particular, .

**Powerset**

The set of all subsets of is called the power set of , denoted

From MAT2612:

Example: is the poset a lattice?

Therefore, is a lattice

Remember that:  
 *number of elements in*

*number of elements in the empty set*

*number of elements in the powerset*

*number of elements in the empty set*

Also:

. *every set is a subset of itself*

**Lesson 0**

<https://math.libretexts.org/Courses/Monroe_Community_College/MATH_220_Discrete_Math/5%3A_Functions/5.2%3A_De%EF%AC%81nition_of_Functions>

Functions

**Function**

Let and be nonempty sets. A function from to is a rule that assigns to every element of a unique element in .

is the domain of the function

is the codomain of the function

If the function is called , we write . Given , its associated element in is called its image.

*for every element in the domain, there exists a unique image in the codomain*

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|  | 0 | 3 | 1 | 4 | 2 | 0 | 3 |

**A Function as a Set of Ordered Pairs**

A function can be written as a set of ordered pairs from such that .

**Identity Function**

The identity function on a nonempty set

Maps any element back to itself

**Inverse function**

Let and be nonempty sets. function is said to be invertible if it has an inverse function.

Notation: If is invertible, we denote the (unique) inverse function by

<https://sites.math.washington.edu/~arms/m300A17/InvFunc.pdf>

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Proof:

Suppose is an invertible function.

Then= a for every ;

= b for every ;

**One-to-One (Injection)**

A function is said to be one-to-one if *an injection*

A function that is not one-to-one is referred to as many-to-one.

To prove a function is One-to-One

1. Assume

2. Show it must be true that

3. Conclude: we have shown if then , therefore is one-to-one, by definition of one-to-one.

To prove a function is NOT One-to-One

1. exhibit one case (a counterexample) where and

2. Conclude: we have shown there is a case where and , therefore is NOT one-to-one.

Example: Prove the function defined by is one-to-one.

Assume , which means .

Thus

so .

We have shown if then . Therefore is one-to-one, by definition of one-to-one

Example: Prove the function defined by is NOT one-to-one.

Consider and .

Clearly .

However, and so .

We have shown there is a case where and , therefore is NOT one-to-one.

**ONTO (Surjection)**

A function is onto if, for every element , there exists an element such that

An onto function is also called a surjection, and we say it is surjective.

Proof:

For

Let be any element in the codomain, .

Figure out an element in the domain that is a preimage of ; often this involves some "scratch work" on the side.

Choose the value you found.

Demonstrate is indeed an element of the domain, .

Show .

Conclude with: we have found a preimage in the domain for an arbitrary element of the codomain, so every element of the codomain has a preimage in the domain. Therefore is onto, by definition of onto.

**One-to-One correspondence**

A function is said to be one-to-one correspondence if and only if is both:

Injective (one-to-one): and,

Surjective (ONTO): for all there is some such that

**Example:** <https://math.libretexts.org/Courses/Monroe_Community_College/MATH_220_Discrete_Math/5%3A_Functions/5.3%3A_One-to-One_Functions>

**Injectivity:**

Take and assume that

Thus

And

So

We have shown if then . Therefore is one-to-one, by definition of one-to-one.

**Surjectivity:**

We need to find an that maps to .

Suppose ;

Now we solve for in terms of .

We find

Proof:

Let be any element of .

Thus, we have found an such that

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<https://math.libretexts.org/Courses/Monroe_Community_College/MATH_220_Discrete_Math/5%3A_Functions/5.4%3A_Onto_Functions_and_Images%2F%2FPreimages_of_Sets>

Onto Functions and Images/Preimages of Sets

**Image of a Set**

Given a function , and , the image of under is defined as

*is the set of all the images of the elements of*

**Preimage of a Set**

Given a function , and , the image of under is defined as

*is the set of all the images of the elements of*